The Small Open Economy in a Generalized Gravity Model

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Motivation

- SOE is a useful first step in analyzing the effects of shocks or policies
- Earlier attempts to consider SOE in new trade models:
  - Flam and Helpman (1987): Krugman
  - Alvarez and Lucas (2007): Eaton–Kortum
  - Demidova and Rodríguez-Clare (2009, 2013): Melitz
- We first revisit the SOE assumption in new trade models:
  - Is there a problem as an economy's labor endowment $\to 0$?
  - If there is, can we fix it to obtain a well-behaved SOE?
- We then use the SOE model to study comp. statics & optimal policy

Contributions

- *General*: a gravity model nesting all standard microfoundations
  - Armington model w/ external economies of scale (EES)
  - Eaton–Kortum model w/ EES
  - Generalized Krugman model w/ nested CES preferences
  - Generalized Melitz–Pareto model w/ nested CES preferences and
    - fixed trade costs paid in labor of destination countries
    - fixed trade costs paid in labor of source countries
  - Kucheryavyy et al. (2022) + tariffs + Melitz–Pareto source case
- *Simple*: only an economy's export & import trade costs are adjusted
  - as an economy's labor endowment $\rightarrow 0$, it becomes "too open"
  - to avoid this, trade should become more costly
- *Intuitive*:
  - graphical approach to comparative statics
  - textbook derivation of optimal tariff formula

Settings

- $N+1$ countries: $i, j = 0, 1, ..., N$ (i: source, j: destination)
- One sector, with varieties differentiated by
  - source countries w/ CES $\eta (> 1)$ (in all but Eaton–Kortum)
  - firms w/ CES $\sigma (\geq \eta)$ (in Krugman, Melitz–Pareto)
- One factor: labor L_i, wage w_i
- Two trade frictions:
  - ad-valorem trade costs $\tau_{ij} (\geq 1), \tau_{jj} = 1$
  - ad-valorem import tariffs $t_{ij} (\geq 1), t_{jj} = 1$

Our generalized gravity model consists of

- Trade shares (expenditure shares j buys from i): λ_{ij}
- Price indices: P_j
- Trade balance (or equivalently, labor market clearing)

Trade shares

$$\lambda_{ij} = \frac{t_{ij}^{-\zeta}[\tau_{ij}/(A_i L_i^{\phi})]^{-\varepsilon} w_i^{-\rho}}{\sum_l t_{lj}^{-\zeta}[\tau_{lj}/(A_l L_l^{\phi})]^{-\varepsilon} w_l^{-\rho}} = \frac{t_{ij}^{-\zeta}(\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l t_{lj}^{-\zeta}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}; \alpha \equiv \varepsilon\phi. \quad (1)$$

- $\varepsilon \equiv -\partial \ln(\lambda_{ij}/\lambda_{jj})/\partial \ln \tau_{ij}$: trade elasticity w.r.t. trade costs
- $\zeta \equiv -\partial \ln(\lambda_{ij}/\lambda_{jj})/\partial \ln t_{ij}$: trade elasticity w.r.t. tariffs
- $\rho \equiv -\partial \ln(\lambda_{ij}/\lambda_{jj})/\partial \ln w_i$: trade elasticity w.r.t. wages
- ϕ: scale elasticity (w/ EES, elasticity of productivity w.r.t. labor)
- $\alpha \equiv \varepsilon\phi$: output elasticity
- $\varepsilon = \zeta = \rho$ in Armington–EES, EK–EES, Gen. Krugman
- $\varepsilon < \zeta$ in Gen. Melitz–Pareto:
  - $\varepsilon = \rho < \zeta$ w/ fixed trade costs paid in labor of destination countries
  - $\varepsilon < \rho = \zeta$ w/ fixed trade costs paid in labor of source countries

Price indices

$$P_j = \delta w_j^{-(\rho/\varepsilon - 1)} [\textstyle\sum_i (\lambda_{ij}/t_{ij})/(L_j/f_{jj})]^{\zeta/\varepsilon - 1} [\textstyle\sum_i t_{ij}^{-\zeta} (\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}]^{-1/\varepsilon}. \tag{2}$$

- δ: model-specific constant
- $P_j = \delta[\sum_i (t_{ij}\tau_{ij} w_i/A_i)^{-\varepsilon} L_i^{\alpha}]^{-1/\varepsilon}$ in Armington–EES, EK–EES, Gen. Krugman
- $[\sum_i (\lambda_{ij}/t_{ij})/(L_j/f_{jj})]^{\zeta/\varepsilon - 1}$: applies only to Gen. Melitz–Pareto
- $w_j^{-(\rho/\varepsilon - 1)}$: applies only to Gen. Melitz–Pareto source

Trade balance

$$w_i L_i = \sum_j \Lambda_{ij} w_j L_j; \qquad (3)$$

$$\Lambda_{ij} \equiv \frac{\lambda_{ij}/t_{ij}}{\sum_l \lambda_{lj}/t_{lj}} = \frac{t_{ij}^{-(\zeta+1)} (\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l t_{lj}^{-(\zeta+1)} (\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}.$$

- (3): i's income = i's revenue, evaluated at pre-tariff import prices
- Λ_{ij}: expenditure share j buys from i, evaluated at pre-tariff import prices

$$\begin{aligned} X_{ij} &= \lambda_{ij} X_j, \\ X_j/(w_j L_j) &= 1/\textstyle\sum_i (\lambda_{ij}/t_{ij}) \\ \Rightarrow X_{ij}/t_{ij} &= [(\lambda_{ij}/t_{ij})/\textstyle\sum_l (\lambda_{lj}/t_{lj})] w_j L_j = \Lambda_{ij} w_j L_j. \end{aligned}$$

- $X_j/(w_j L_j) = 1/\sum_i (\lambda_{ij}/t_{ij}) (\geq 1)$: j's tariff multiplier[1]

[1] Felbermayr et al. (2015)

Equilibrium

Definition

An equilibrium is a wage vector $w \equiv (w_0, w_1, ..., w_N)$ such that (3) holds for all i.

Proposition

There exists a unique equilibrium.

Proof.

Just invoke MWG's existence and uniqueness propositions! □

Microfoundations: Armington–EES

$$p_{ij} = w_i/(\overline{A}_i L_i^{\gamma}).$$

- $\overline{A}_i$: constant productivity
- γ: elasticity of productivity w.r.t. labor as EES

$$\lambda_{ij} = \frac{(t_{ij}\overline{\tau}_{ij} w_i/\overline{A}_i)^{-(\eta-1)} L_i^{(\eta-1)\gamma}}{\sum_l (t_{lj}\overline{\tau}_{lj} w_l/\overline{A}_l)^{-(\eta-1)} L_l^{(\eta-1)\gamma}},$$

$$P_j = [\textstyle\sum_i (t_{ij}\overline{\tau}_{ij} w_i/\overline{A}_i)^{-(\eta-1)} L_i^{(\eta-1)\gamma}]^{-1/(\eta-1)}.$$

- $\overline{\tau}_{ij}(\geq 1)$: iceberg trade cost of selling from i to j
  - of $\overline{\tau}_{ij}$ units shipped in i, $\overline{\tau}_{ij} - 1$ units melt away, 1 unit arrives at j

Microfoundations: EK–EES

$$p_{ij}(\omega) = w_i/(\varphi L_i^{\gamma}), \omega \in [0,1].$$

- φ: random productivity $\sim$ Frechet (ϑ, B_i)
- Cumulative distribution: $G_i(\varphi) \equiv \exp(-B_i \varphi^{-\vartheta}); B_i > 0, \vartheta > \sigma - 1$

$$\lambda_{ij} = \frac{B_i (t_{ij} \overline{\tau}_{ij} w_i / L_i^{\gamma})^{-\vartheta}}{\Phi_j}; \Phi_j \equiv \textstyle\sum_l B_l (t_{lj} \overline{\tau}_{lj} w_l / L_l^{\gamma})^{-\vartheta},$$

$$P_j = \delta^{EK} \Phi_j^{-1/\vartheta} = \delta^{EK} [\textstyle\sum_i B_i (t_{ij} \overline{\tau}_{ij} w_i / L_i^{\gamma})^{-\vartheta}]^{-1/\vartheta};$$

$$\delta^{EK} \equiv \Gamma(1 + (1-\sigma)/\vartheta)^{1/(1-\sigma)}.$$

- $\Gamma(\cdot)$: Gamma function

Microfoundations: Gen. Krugman

$$p_{ij}(\omega) = w_i/(\mu a_i); \mu \equiv 1 - 1/\sigma \in (0,1).$$

- a_i: constant productivity
- μ: inverse of constant markup

$$\lambda_{ij} = \frac{[t_{ij}\overline{\tau}_{ij} w_i/((f_i^e)^{-1/(\sigma-1)} a_i)]^{-(\eta-1)} L_i^{(\eta-1)/(\sigma-1)}}{\sum_l [t_{lj}\overline{\tau}_{lj} w_l/((f_l^e)^{-1/(\sigma-1)} a_l)]^{-(\eta-1)} L_l^{(\eta-1)/(\sigma-1)}},$$

$$P_j = \delta^K \{\textstyle\sum_i [t_{ij}\overline{\tau}_{ij} w_i/((f_i^e)^{-1/(\sigma-1)} a_i)]^{-(\eta-1)} L_i^{(\eta-1)/(\sigma-1)}\}^{-1/(\eta-1)};$$

$$\delta^K \equiv \sigma^{1/(\sigma-1)} \mu^{-1}.$$

- f_i^e: fixed entry cost in terms of i's labor

Microfoundations: Gen. Melitz–Pareto destination

$$p_{ij}(\varphi) = w_i/(\mu\varphi).$$

- φ: random productivity $\sim$ Pareto (b_i, θ)
- Cumulative distribution: $G_i(\varphi) \equiv 1 - b_i^{\theta}\varphi^{-\theta}; \varphi \in [b_i, \infty), \theta > \sigma - 1$

$$\lambda_{ij} = \frac{t_{ij}^{-\theta\xi\iota}[(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij}/((f_i^e)^{-1/\theta}b_i)]^{-\theta\xi}w_i^{-\theta\xi}L_i^{\xi}}{\sum_l t_{lj}^{-\theta\xi\iota}[(f_{lj}/f_{jj})^{\iota-1}\overline{\tau}_{lj}w_l/((f_l^e)^{-1/\theta}b_l)]^{-\theta\xi}w_l^{-\theta\xi}L_l^{\xi}};$$

$$\xi \equiv \{1 + \theta[1/(\eta-1) - 1/(\sigma-1)]\}^{-1} \in [0,1],$$

$$\iota \equiv 1 + 1/(\sigma-1) - 1/\theta > 1,$$

$$P_j = \delta^M \left[\frac{\sum_i(\lambda_{ij}/t_{ij})}{L_j/f_{jj}}\right]^{\iota-1}\left\{\sum_i t_{ij}^{-\theta\xi\iota}\left[\frac{(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij}}{(f_i^e)^{-1/\theta}b_i}\right]^{-\theta\xi} w_i^{-\theta\xi}L_i^{\xi}\right\}^{-1/(\theta\xi)};$$

$$\delta^M \equiv (\theta/\kappa - 1)^{-1/\theta}\delta^K.$$

- f_{ij}: fixed trade cost of selling from i to j in terms of j's labor
- $\xi = 1$ for $\sigma = \eta$ (standard Melitz–Pareto); $\xi \downarrow$ as $\sigma \uparrow$ and/or $\eta \downarrow$

Q1: Why is λ_{ij} more elastic w.r.t. t_{ij} than w.r.t. $\overline{\tau}_{ij}$?

Firm φ's revenue of selling from i to j:

$$r_{ij}(\varphi) = p_{ij}(\varphi)\overline{\tau}_{ij} \times (t_{ij}\overline{\tau}_{ij}p_{ij}(\varphi))^{-\sigma} \times \text{aggregate demand term}$$

$$\Rightarrow -\partial \ln r_{ij}(\varphi)/\partial \ln t_{ij} = \sigma > \sigma - 1 = -\partial \ln r_{ij}(\varphi)/\partial \ln \overline{\tau}_{ij}.$$

Zero cutoff profit (ZCP) condition:

$$\pi_{ij}(\varphi_{ij}) = 0 \Leftrightarrow r_{ij}(\varphi_{ij}) = t_{ij}^{-\sigma}[\overline{\tau}_{ij}w_i/(\mu\varphi_{ij}P_{ij})]^{1-\sigma}(P_{ij}/P_j)^{1-\eta}X_j = \sigma w_j f_{ij}.$$

- φ_{ij}: cutoff productivity of selling from i to j ($\pi_{ij}(\varphi) \geq 0 \Leftrightarrow \varphi \geq \varphi_{ij}$)
- P_{ij}: j's price index for varieties from i

Relative cutoff condition:

$$\frac{t_{ij}^{-\sigma}[\overline{\tau}_{ij}w_i/(\mu\varphi_{ij}P_{ij})]^{1-\sigma}(P_{ij}/P_j)^{1-\eta}X_j}{[w_j/(\mu\varphi_{jj}P_{jj})]^{1-\sigma}(P_{jj}/P_j)^{1-\eta}X_j} = \frac{\sigma w_j f_{ij}}{\sigma w_j f_{jj}}$$

$$\Leftrightarrow \varphi_{ij}/\varphi_{jj} = t_{ij}^{\sigma/(\sigma-1)}(\overline{\tau}_{ij}w_i/w_j)(f_{ij}/f_{jj})^{1/(\sigma-1)}(P_{ij}/P_{jj})^{(\eta-1)/(\sigma-1)-1}$$

$$\Rightarrow \partial \ln(\varphi_{ij}/\varphi_{jj})/\partial \ln t_{ij} = \sigma/(\sigma-1) > 1 = \partial \ln(\varphi_{ij}/\varphi_{jj})/\partial \ln \overline{\tau}_{ij}.$$

$\therefore$ $\varphi_{ij}/\varphi_{jj}$ is more elastic w.r.t. t_{ij} than w.r.t. $\overline{\tau}_{ij}$

Q2: What is $[\sum_i(\lambda_{ij}/t_{ij})/(L_j/f_{jj})]^{\iota-1}$ in P_j?

Domestic ZCP condition, tariff multiplier $\rightarrow$ domestic cutoff:

$$[w_j/(\mu\varphi_{jj}P_{jj})]^{1-\sigma}(P_{jj}/P_j)^{1-\eta}X_j = \sigma w_j f_{jj},$$
$$X_j/(w_j L_j) = 1/\sum_i(\lambda_{ij}/t_{ij})$$
$$\Rightarrow \varphi_{jj} = \sigma^{1/(\sigma-1)}\mu^{-1}[(L_j/f_{jj})/\sum_i(\lambda_{ij}/t_{ij})]^{-1/(\sigma-1)} w_j P_{jj}^{(\eta-1)/(\sigma-1)-1} P_j^{-(\eta-1)/(\sigma-1)}.$$

- $(L_j/f_{jj})/\sum_i(\lambda_{ij}/t_{ij})$ captures j's market size (incl. tariff multiplier)
- $(L_j/f_{jj})/\sum_i(\lambda_{ij}/t_{ij}) \uparrow \rightarrow \varphi_{jj} \downarrow$: larger mkt accomodates weaker firms
- $\varphi_{jj} \downarrow \rightarrow M_j^e(1 - G_j(\varphi_{jj})) \uparrow \rightarrow P_j \downarrow \because$ more surviving dom entrants
- $\theta \uparrow \rightarrow \iota - 1 = 1/(\sigma - 1) - 1/\theta \uparrow \rightarrow$ above effect is stronger

Microfoundations: Gen. Melitz–Pareto source

Only different assumption from destination case: f_{ij} is in terms of i's labor

$$\lambda_{ij} = \frac{t_{ij}^{-\theta\xi\iota}[(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij}/((f_i^e)^{-1/\theta}b_i)]^{-\theta\xi}w_i^{-\theta\xi\iota}L_i^{\xi}}{\sum_l t_{lj}^{-\theta\xi\iota}[(f_{lj}/f_{jj})^{\iota-1}\overline{\tau}_{lj}w_l/((f_l^e)^{-1/\theta}b_l)]^{-\theta\xi}w_l^{-\theta\xi\iota}L_l^{\xi}};$$

$$\xi \equiv \{1+\theta[1/(\eta-1)-1/(\sigma-1)]\}^{-1} \in [0,1],$$

$$\iota \equiv 1+1/(\sigma-1)-1/\theta > 1,$$

$$P_j = \delta^M w_j^{-(\iota-1)}\left[\frac{\sum_i(\lambda_{ij}/t_{ij})}{L_j/f_{jj}}\right]^{\iota-1}\left\{\sum_i t_{ij}^{-\theta\xi\iota}\left[\frac{(f_{ij}/f_{jj})^{\iota-1}\overline{\tau}_{ij}}{(f_i^e)^{-1/\theta}b_i}\right]^{-\theta\xi} w_i^{-\theta\xi\iota}L_i^{\xi}\right\}^{-1/(\theta\xi)};$$

$$\delta^M \equiv (\theta/\kappa-1)^{-1/\theta}\delta^K.$$

Compared with destination case:

- $\partial \ln(\lambda_{ij}/\lambda_{jj})/\partial \ln w_i$ increases from $\theta\xi$ to $\theta\xi\iota$
- $w_j^{-(\iota-1)}$ is added to P_j (so P_j is linearly homogeneous w.r.t. $\{w_i\}_{i=0}^N$)

Q1: Why is λ_{ij} more elastic w.r.t. w_i than dest. case?

Relative cutoff condition:

$$\frac{t_{ij}^{-\sigma}[\overline{\tau}_{ij} w_i/(\mu \varphi_{ij} P_{ij})]^{1-\sigma}(P_{ij}/P_j)^{1-\eta} X_j}{[w_j/(\mu \varphi_{jj} P_{jj})]^{1-\sigma}(P_{jj}/P_j)^{1-\eta} X_j} = \frac{\sigma w_i f_{ij}}{\sigma w_j f_{jj}}$$
$$\Leftrightarrow \varphi_{ij}/\varphi_{jj} = (t_{ij} w_i/w_j)^{\sigma/(\sigma-1)} \overline{\tau}_{ij} (f_{ij}/f_{jj})^{1/(\sigma-1)} (P_{ij}/P_{jj})^{(\eta-1)/(\sigma-1)-1}$$
$$\Rightarrow \partial \ln(\varphi_{ij}/\varphi_{jj})/\partial \ln w_i = \sigma/(\sigma-1) > 1.$$

$w_i/w_j \uparrow \rightarrow \varphi_{ij}/\varphi_{jj} \uparrow$ (less exporters from i sell to j) via two channels:

- Relative gross profit of exporters from $i \downarrow$ (also in destination case)
- Relative fixed trade cost of exporters from $i \uparrow$ (new in source case)

$\therefore$ $\varphi_{ij}/\varphi_{jj}$ is more elastic w.r.t. w_i than destination case

Microfoundations: summary

Model	Armington–EES	EK–EES	Gen. Krugman	Gen. Melitz destination	Gen. Melitz source
ε	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi$
ζ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi[1 + 1/(\sigma - 1) - 1/\theta]$	$\theta\xi[1 + 1/(\sigma - 1) - 1/\theta]$
ρ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi[1 + 1/(\sigma - 1) - 1/\theta]$
ϕ	γ	γ	$1/(\sigma - 1)$	$1/\theta$	$1/\theta$
$\alpha \equiv \varepsilon\phi$	$(\eta - 1)\gamma$	$\vartheta\gamma$	$(\eta - 1)/(\sigma - 1)$	ξ	ξ
τ_{ij}	$\overline{\tau}_{ij}$	$\overline{\tau}_{ij}$	$\overline{\tau}_{ij}$	$(f_{ij}/f_{jj})^{1/(\sigma-1)-1/\theta}\overline{\tau}_{ij}$	$(f_{ij}/f_{jj})^{1/(\sigma-1)-1/\theta}\overline{\tau}_{ij}$
A_i	$\overline{A}_i$	$B_i^{1/\vartheta}$	$(f_i^e)^{-1/(\sigma-1)}a_i$	$(f_i^e)^{-1/\theta}b_i$	$(f_i^e)^{-1/\theta}b_i$
δ	1	δ^{EK}	δ^{K}	δ^{M}	δ^{M}

Table 1: Mapping the five different trade models into the general model

- $\varepsilon = \zeta = \rho$ in Armington–EES, EK–EES, Gen. Krugman
- $\varepsilon < \zeta$ in Gen. Melitz–Pareto:
  - $\varepsilon = \rho < \zeta$ in destination case
  - $\varepsilon < \rho = \zeta$ in source case

Welfare

(1) for $i = j$, (2) $\rightarrow$ real wage:

$$w_j/P_j = [(L_j/f_{jj})/\sum_i(\lambda_{ij}/t_{ij})]^{\zeta/\varepsilon-1}\delta^{-1}A_jL_j^{\phi}\lambda_{jj}^{-1/\varepsilon}.$$

Welfare (real expenditure per capita $X_j/(P_jL_j)$):

$$W_j \equiv [1/\sum_i(\lambda_{ij}/t_{ij})](w_j/P_j) = (L_j/f_{jj})^{\zeta/\varepsilon-1}[1/\sum_i(\lambda_{ij}/t_{ij})]^{\zeta/\varepsilon}\delta^{-1}A_jL_j^{\phi}\lambda_{jj}^{-1/\varepsilon}.$$

In autarky ($\lambda_{jj} = 1, \lambda_{ij} = 0 \forall i \neq j$):

$$W_j = (L_j/f_{jj})^{\zeta/\varepsilon-1}\delta^{-1}A_jL_j^{\phi} \equiv W_j^A.$$

Gains from trade:

$$\mathrm{GT}_j \equiv W_j/W_j^A \equiv \lambda_{jj}^{-1/\varepsilon}[\sum_i(\lambda_{ij}/t_{ij})]^{-\zeta/\varepsilon}.$$

- w/ tariffs, ε and λ_{jj} are not sufficient to quantify GT_j
- GT_j is different btw Gen. Melitz–Pareto ($\zeta > \varepsilon$) and the rest ($\zeta = \varepsilon$)

A first look

Generalized gravity model:

$$\lambda_{ij} = \frac{t_{ij}^{-\zeta}(\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l t_{lj}^{-\zeta}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}},$$

$$\Lambda_{ij} \equiv \frac{\lambda_{ij}/t_{ij}}{\sum_l \lambda_{lj}/t_{lj}} = \frac{t_{ij}^{-(\zeta+1)}(\tau_{ij}/A_i)^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{\sum_l t_{lj}^{-(\zeta+1)}(\tau_{lj}/A_l)^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}},$$

$$w_i L_i = \sum_j \Lambda_{ij} w_j L_j.$$

Country 0's labor endowment:

$$L_0 \equiv n\widetilde{L}_0.$$

As $n \to 0$, country 0 becomes a SOE in the limit

To understand potential problems of a SOE eq., we first consider:

- Two countries: $i, j = 0, 1$
- Two popular examples:
  - Armington or EK w/o EES ($\alpha = 0$)
  - standard Krugman or Melitz–Pareto ($\alpha = 1$)

Armington or EK w/o EES: Problem

0's trade balance (TB) ($w_1 \equiv 1$):

$$\underbrace{\frac{A_0^{\varepsilon} t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} w_0^{-\rho}}{A_0^{\varepsilon} t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} w_0^{-\rho} + A_1^{\varepsilon}}}_{=\Lambda_{01}} L_1 = \underbrace{\frac{A_1^{\varepsilon} t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}}{A_0^{\varepsilon} w_0^{-\rho} + A_1^{\varepsilon} t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}}}_{=\Lambda_{10}} w_0 n \widetilde{L}_0.$$

- LHS: 0's exports (1's imports), decreasing in w_0
- RHS: 0's imports, increasing in w_0
- $\forall n > 0$, there exists a unique eq. $w_0 \in (0, \infty)$ satisfying 0's TB
- $n \downarrow \to$ RHS $\downarrow$ for each $w_0 \to w_0 \uparrow$
- Problem: as $n \to 0$, $w_0 \to \infty \Rightarrow \Lambda_{10} \to 1 \Rightarrow \Lambda_{00} \to 0$
- $\Lambda_{00} \to 0$ prevents us from mapping model to data (where $\Lambda_{00} > 0$)
- Reason: as $n \to 0$, 0's import demand $\to 0$, which leads to $w_0 \to \infty$
- Idea: what if 0's exports become more and more costly as $n \downarrow$?

Armington or EK w/o EES: Solution

Adjustment in τ_{01} (0's exports become more and more costly as $n \downarrow$):

$$\tau_{01} \equiv n^{-1/\varepsilon} \widetilde{\tau}_{01}.$$

- Substitute this into 0's TB
- Multiply both sides by $1/n$
- Let $n \to 0$

$$\frac{A_0^{\varepsilon} t_{01}^{-1-\zeta} \widetilde{\tau}_{01}^{-\varepsilon} \widetilde{w}_0^{-\rho}}{A_1^{\varepsilon}} L_1 = \frac{A_1^{\varepsilon} t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}}{A_0^{\varepsilon} \widetilde{w}_0^{-\rho} + A_1^{\varepsilon} t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon}} \widetilde{w}_0 \widetilde{L}_0.$$

$\therefore$ There exists a unique eq. $\widetilde{w}_0 \in (0, \infty)$ satisfying 0's TB $\Rightarrow \Lambda_{00} \in (0, 1)$

Standard Krugman or Melitz–Pareto: Problem

0's TB ($w_1 \equiv 1$):

$$\underbrace{\frac{A_0^\varepsilon t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} w_0^{-\rho} n \widetilde{L}_0}{A_0^\varepsilon t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} w_0^{-\rho} n \widetilde{L}_0 + A_1^\varepsilon L_1}}_{=\Lambda_{01}} L_1 = \underbrace{\frac{A_1^\varepsilon t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon} L_1}{A_0^\varepsilon w_0^{-\rho} n \widetilde{L}_0 + A_1^\varepsilon t_{10}^{-1-\zeta} \tau_{10}^{-\varepsilon} L_1}}_{=\Lambda_{10}} w_0 n \widetilde{L}_0.$$

- Problem: as $n \to 0$, $\Lambda_{10} \to 1 \Rightarrow \Lambda_{00} \to 0$ even if $w_0 \in (0, \infty)$
- Reason: as $n \to 0$, 0's mass of entrants ($\propto$ 0's labor endowment) $\to 0$
- Idea: what if 0's imports become more and more costly as $n \downarrow$?

Standard Krugman or Melitz–Pareto: Solution

Adjustment in τ_{10} (0's imports become more and more costly as $n \downarrow$):

$$\tau_{10} \equiv n^{-1/\varepsilon} \widetilde{\tau}_{10}.$$

- Substitute this into 0's TB
- Multiply both sides by $1/n$
- Let $n \to 0$

$$\frac{A_0^{\varepsilon} t_{01}^{-1-\zeta} \tau_{01}^{-\varepsilon} \widetilde{w}_0^{-\rho} \widetilde{L}_0}{A_1^{\varepsilon} L_1} L_1 = \frac{A_1^{\varepsilon} t_{10}^{-1-\zeta} \widetilde{\tau}_{10}^{-\varepsilon} L_1}{A_0^{\varepsilon} \widetilde{w}_0^{-\rho} \widetilde{L}_0 + A_1^{\varepsilon} t_{10}^{-1-\zeta} \widetilde{\tau}_{10}^{-\varepsilon} L_1} \widetilde{w}_0 \widetilde{L}_0.$$

$\therefore$ There exists a unique eq. $\widetilde{w}_0 \in (0,1)$ satisfying 0's TB $\Rightarrow \Lambda_{00} \in (0,1)$

General case

Adjustment rule for $N+1$ countries and a general α:

$$\begin{aligned} L_0 &\equiv n\widetilde{L}_0, \\ \tau_{0j} &\equiv n^{-(1-\alpha)/\varepsilon}\widetilde{\tau}_{0j}, j = 1, ..., N, \\ \tau_{i0} &\equiv n^{-\alpha/\varepsilon}\widetilde{\tau}_{i0}, i = 1, ..., N. \end{aligned}$$

- Includes the two examples as extreme cases:
  - Armington or EK w/o EES ($\alpha = 0$): $\tau_{0j} \equiv n^{-1/\varepsilon}\widetilde{\tau}_{0j}, \tau_{i0} \equiv \widetilde{\tau}_{i0}$
  - standard Krugman or Melitz–Pareto ($\alpha = 1$): $\tau_{0j} \equiv \widetilde{\tau}_{0j}, \tau_{i0} \equiv n^{-1/\varepsilon}\widetilde{\tau}_{i0}$
- As $\alpha \uparrow$:
  - adjust 0's export trade costs less and less
  - adjust 0's import trade costs more and more

SOE Equilibrium

Proposition

As country 0 becomes arbitrary small (i.e., as $n \to 0$), the equilibrium converges to the one in which:

1. $(w_1, ..., w_N)$ solves (3) for all $i, j, l = 1, ..., N$ not including country 0;

2. $\widetilde{w}_0$ solves

$$\sum_{j=1}^{N} \frac{A_0^{\varepsilon} t_{0j}^{-1-\zeta} \widetilde{\tau}_{0j}^{-\varepsilon} \widetilde{w}_0^{-\rho} \widetilde{L}_0^{\alpha}}{\sum_{i=1}^{N} A_i^{\varepsilon} t_{ij}^{-1-\zeta} \tau_{ij}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}} w_j L_j = \frac{\sum_{i=1}^{N} A_i^{\varepsilon} t_{i0}^{-1-\zeta} \widetilde{\tau}_{i0}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}{A_0^{\varepsilon} \widetilde{w}_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{i=1}^{N} A_i^{\varepsilon} t_{i0}^{-1-\zeta} \widetilde{\tau}_{i0}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}} \widetilde{w}_0 \widetilde{L}_0. \tag{4}$$

Sketch of proof of Proposition 2

- Rewrite trade shares and trade balances using adjustment rule
- As $n \to 0$, TB for $i = 1, ..., N$ reduce to (3), not including country 0
  $\Rightarrow$ there exists a unique $(w_1, ..., w_N)$, independent of country 0
- TB for $i = 0$:

$$w_0 n\widetilde{L}_0 = \Lambda_{00} w_0 n\widetilde{L}_0 + \textstyle\sum_{j=1}^{N} \Lambda_{0j} w_j L_j$$
$$w_0 \widetilde{L}_0 = \Lambda_{00} w_0 \widetilde{L}_0 + \textstyle\sum_{j=1}^{N} (\Lambda_{0j}/n) w_j L_j.$$

- How is Λ_{0j}/n like?

$$\Lambda_{0j} = \frac{A_0^{\varepsilon} t_{0j}^{-1-\zeta} (n^{-(1-\alpha)/\varepsilon} \widetilde{\tau}_{0j})^{-\varepsilon} w_0^{-\rho} (n\widetilde{L}_0)^{\alpha}}{A_0^{\varepsilon} t_{0j}^{-1-\zeta} (n^{-(1-\alpha)/\varepsilon} \widetilde{\tau}_{0j})^{-\varepsilon} w_0^{-\rho} (n\widetilde{L}_0)^{\alpha} + \sum_{l=1}^{N} A_l^{\varepsilon} t_{lj}^{-1-\zeta} \tau_{lj}^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}}$$
$$= \frac{n A_0^{\varepsilon} t_{0j}^{-1-\zeta} \widetilde{\tau}_{0j}^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha}}{n A_0^{\varepsilon} t_{0j}^{-1-\zeta} \widetilde{\tau}_{0j}^{-\varepsilon} w_0^{-\rho} \widetilde{L}_0^{\alpha} + \sum_{l=1}^{N} A_l^{\varepsilon} t_{lj}^{-1-\zeta} \tau_{lj}^{-\varepsilon} w_l^{-\rho} L_l^{\alpha}} \equiv n\widetilde{\Lambda}_{0j}.$$

- Since $\widetilde{\Lambda}_{0j} \equiv \Lambda_{0j}/n > 0$ even if $n \to 0$, TB for $i = 0$ implies (4):

$$\textstyle\sum_{j=1}^{N} \widetilde{\Lambda}_{0j} w_j L_j = (1 - \widetilde{\Lambda}_{00}) \widetilde{w}_0 \widetilde{L}_0; \widetilde{\Lambda}_{00} \equiv \Lambda_{00}.$$

Implications of Proposition 2

- There exists a unique eq. $\widetilde{w}_0 \in (0, \infty)$ solving (4) $\Rightarrow \widetilde{\Lambda}_{00} \in (0, 1)$, which allows us to map model to data
- All variables w/in countries 1 to N are determined independently of 0 $\Rightarrow$ a limit-economy foundation for DRC (2009, 2013)

SOE model with simplified notation

Omitting tildes and subindices "0", SOE's TB (4) is simplified to:

$$X(w) = M(w); \tag{11}$$

$$X(w) \equiv DA^{\varepsilon} L^{\alpha} w^{-\rho},$$

$$D \equiv \sum_{j=1}^{N} \frac{\tau_{*j}^{-\varepsilon} w_j L_j}{\sum_{i=1}^{N} A_i^{\varepsilon} t_{ij}^{-(\zeta+1)} \tau_{ij}^{-\varepsilon} w_i^{-\rho} L_i^{\alpha}}, \tau_{*j} \equiv t_{0j}^{(\zeta+1)/\varepsilon} \widetilde{\tau}_{0j},$$

$$M(w) \equiv \frac{\sum_{i=1}^{N} \lambda_i(w)/t_i}{1 - \sum_{i=1}^{N} (1 - 1/t_i) \lambda_i(w)} wL,$$

$$\lambda_i(w) \equiv \frac{t_i^{-\zeta} p_i^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + \sum_{l=1}^{N} t_l^{-\zeta} p_l^{-\rho}}, p_i \equiv [\tau_{i*}/(A_i L_i^{\phi})]^{\varepsilon/\rho} w_i, \tau_{i*} \equiv \widetilde{\tau}_{i0}.$$

- $X(w)$: SOE's exports (RoW's imports), decreasing in w
  - D: SOE's export market access, exogenous
- $M(w)$: SOE's imports, increasing in w
  - $\lambda_i(w)$: SOE's import expenditure share from i
  - p_i: SOE's price of imports from i, exogenous

SOE's gains from trade:

$$\mathrm{GT} \equiv W/W^A \equiv \lambda^{-1/\varepsilon} [\lambda + \sum_{i=1}^{N} (\lambda_i/t_i)]^{-\zeta/\varepsilon}. \tag{12}$$

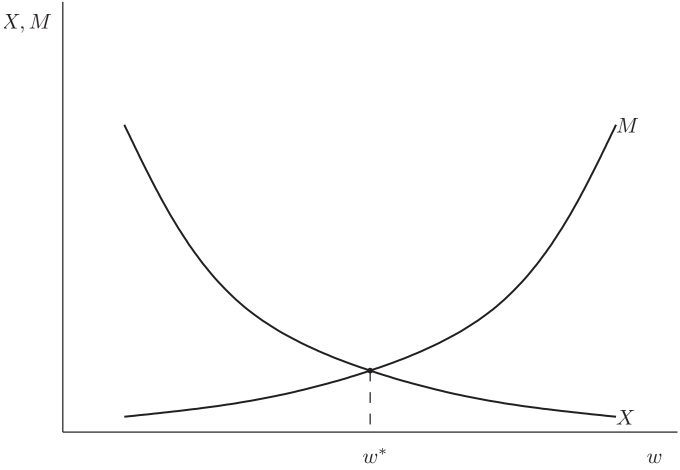
Comparative statics

$t_i = t \forall i$ (no tariff-induced distortion across import sources) $\Rightarrow$

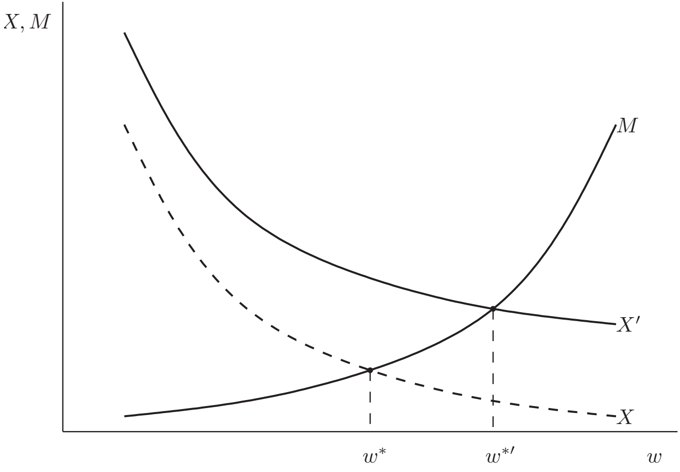
$$X(w) = M(w);$$
$$X(w) \equiv DA^{\varepsilon} L^{\alpha} w^{-\rho},$$
$$M(w) \equiv \frac{1-\lambda(w)}{1+(t-1)\lambda(w)} wL,$$
$$\lambda(w) \equiv \frac{A^{\varepsilon} L^{\alpha} w^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + t^{-\zeta}\mathcal{P}^{-\rho}}, \mathcal{P}^{-\rho} \equiv \sum_{i=1}^{N} p_i^{-\rho},$$
$$\mathrm{GT} = \lambda(w)^{-1/\varepsilon}[\lambda(w) + (1-\lambda(w))/t]^{-\zeta/\varepsilon}.$$

- $X(w)$: SOE's exports (RoW's imports), decreasing in w
  - D: SOE's export market access, exogenous
- $M(w)$: SOE's imports, increasing in w
  - $\lambda(w)$: SOE's domestic trade share
  - $\mathcal{P}$: SOE's import market access, exogenous
- GT: SOE's gains from trade

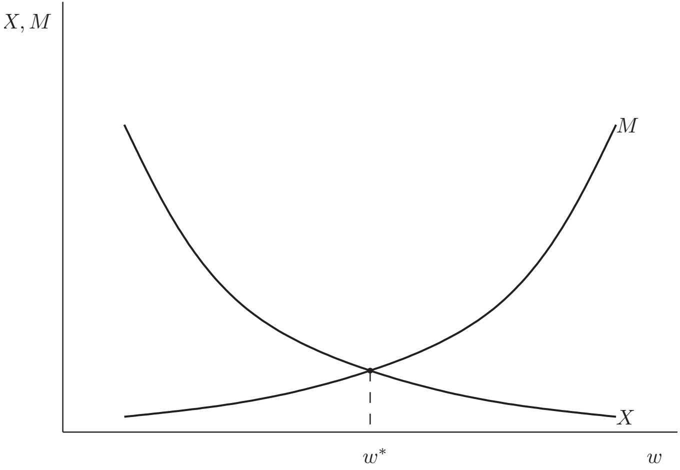
An increase in D increases w, increases $X = M$



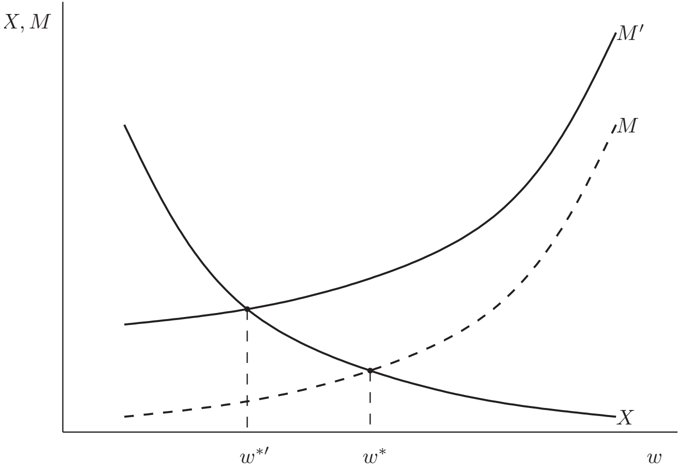
An increase in D increases w, increases $X = M$



A decrease in $\mathcal{P}$ decreases w, increases $X = M$



A decrease in $\mathcal{P}$ decreases w, increases $X = M$



Summary on comparative statics

$$
\begin{aligned}
X(w) &= M(w); \\
X(w) &\equiv DA^{\varepsilon} L^{\alpha} w^{-\rho}, \\
M(w) &\equiv \frac{1-\lambda(w)}{1+(t-1)\lambda(w)} wL, \\
\lambda(w) &\equiv \frac{A^{\varepsilon} L^{\alpha} w^{-\rho}}{A^{\varepsilon} L^{\alpha} w^{-\rho} + t^{-\zeta}\mathcal{P}^{-\rho}}, \\
\mathrm{GT} &= \lambda(w)^{-\varepsilon}[\lambda(w) + (1-\lambda(w))/t]^{-\zeta/\varepsilon}.
\end{aligned}
$$

$D \uparrow \rightarrow$:

- $X > M \rightarrow w \uparrow$ to restore trade balance $\rightarrow X = M \uparrow$
- $w \uparrow \rightarrow \lambda \downarrow \rightarrow \mathrm{GT} \uparrow$

$\mathcal{P} \downarrow \rightarrow$:

- $X < M \rightarrow w \downarrow$ to restore trade balance $\rightarrow X = M \uparrow$
- for $M \uparrow$ in spite of $w \downarrow$, we need $\lambda \downarrow$, so $\mathrm{GT} \uparrow$

$\therefore$ improved markt access in export or import increases SOE's trade, welfare

Effect of SOE's tariff increase on its terms of trade

Log differentiate SOE's TB (11) ($\widehat{x} \equiv d \ln x$) $\rightarrow$ effect of t_i on w:

$$\frac{\partial \ln w}{\partial \ln t_i} = \frac{\zeta + 1}{\Delta} \frac{\Lambda}{1 - \Lambda} \Lambda_i > 0; \qquad (13)$$

$$\Delta \equiv 1 + \rho(1 + \Lambda) > 1.$$

- $t_i \uparrow \rightarrow X > M \rightarrow w \uparrow$ to restore trade balance $\rightarrow X = M \downarrow$
- Just the opposite of $\mathcal{P} \downarrow$
- Since labor is the only primary factor, $w \uparrow \rightarrow$ SOE's ToT $\uparrow$

Effect of SOE's tariff increase on its gains from trade

Log differentiate SOE's GT (12), use (13) $\rightarrow$ effect of t_i on GT:

$$\frac{\partial \ln \mathrm{GT}}{\partial \ln t_i} = \underbrace{(\rho/\varepsilon)[1-\lambda+\zeta(\Lambda-\lambda)]\frac{\zeta+1}{\Delta}\frac{\Lambda}{1-\Lambda}\Lambda_i}_{\text{terms of trade effect}} \underbrace{-(\zeta/\varepsilon)(\zeta+1)(\lambda_i-\Lambda_i)}_{\text{distortionary effect}}. \tag{14}$$

- $t_i \uparrow \rightarrow$:
  - $(+)$ terms of trade effect: $w \uparrow \rightarrow \lambda \downarrow \rightarrow \mathrm{GT} \uparrow$
  - $(-)$ distortionary effect: aggravates under-imports $\rightarrow \mathrm{GT} \downarrow$
- $\rho \uparrow \rightarrow$ |terms of trade effect| $\uparrow \rightarrow$ incentive to raise $t_i \uparrow$
- $\zeta \uparrow \rightarrow$ |distortionary effect| $\uparrow \rightarrow$ incentive to raise $t_i \downarrow$
- SOE's optimal tariff on imports from i, t_i^*:

$$\frac{\partial \ln \mathrm{GT}}{\partial \ln t_i}\Big|_{t_i=t_i^*} = 0,$$

or, |terms of trade effect| = |distortionary effect|

SOE's optimal tariff formula

Proposition

The SOE's optimal tariff is given by

$$t^* - 1 = 1/[(1+\rho)(\zeta/\rho) - 1] \forall i = 1, ..., N. \tag{15}$$

- $t^* - 1 > 0$ in contrast to textbook SOE (where $t^* - 1 = 0$)
- $t_i^* = t^* \forall i$ ($\because$ diff. across i affects ToT & distortionary effects equally)
- t^* is increasing in ρ (given ζ), decreasing in ζ (given ρ), as expected

Further implications of SOE's optimal tariff formula

$$t^* - 1 = 1/[(1+\rho)(\zeta/\rho) - 1] \forall i = 1, ..., N.$$

Model	Armington–EES	EK–EES	Gen. Krugman	Gen. Melitz destination	Gen. Melitz source
ε	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi$
ζ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi[1 + 1/(\sigma-1) - 1/\theta]$	$\theta\xi[1 + 1/(\sigma-1) - 1/\theta]$
ρ	$\eta - 1$	ϑ	$\eta - 1$	$\theta\xi$	$\theta\xi[1 + 1/(\sigma-1) - 1/\theta]$
$t^* - 1$	$\frac{1}{\eta-1}$	$\frac{1}{\vartheta}$	$\frac{1}{\eta-1}$	$\frac{1}{(1+\theta\xi)[1+1/(\sigma-1)-1/\theta]-1}$	$\frac{1}{\theta\xi[1+1/(\sigma-1)-1/\theta]}$

Table 2: The SOE's optimal tariff for the five microfoundations

- $t^* - 1 = 1/\rho = 1/\varepsilon$ in Armington–EES, EK–EES, Gen. Krugman
- $t^* - 1 = 1/\rho < 1/\varepsilon$ in Gen. Melitz–Pareto source
- $t^* - 1 < 1/\rho$ in Gen. Melitz–Pareto destination
- For Gen. Melitz–Pareto, t^* in destination case $<$ t^* in source case:
  - for common ρ: ζ larger in destination case
  - for common ε and ζ: ρ smaller in destination case
  - $\because$ compared with source case, in destination case $t \uparrow \rightarrow$ RoW's exports $\downarrow \rightarrow$ SOE's L demand $\downarrow \rightarrow$ weaker ToT effect

Conclusion

What we got:

- *General* gravity model nesting all standard microfoundations
- *Simple* trade cost adjustment rule for a well-behaved SOE
- *Intuitive* comparative statics and optimal tariff formula